Wavelet-Powered Neural Networks for Turbulence

Dr. Arvind T. Mohan

Postdoctoral Researcher

Center for Nonlinear Studies

Computational Physics & Methods Group

Los Alamos National Laboratory, New Mexico

Valles Caldera National Preserve

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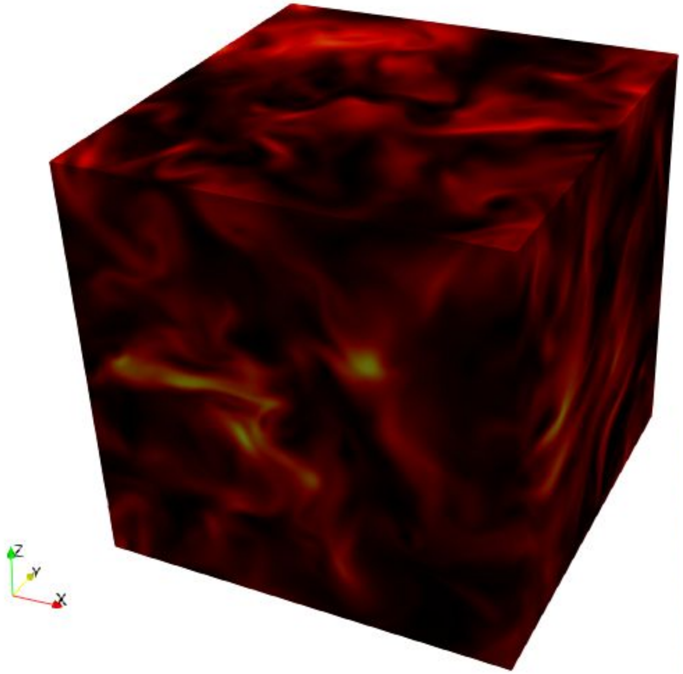
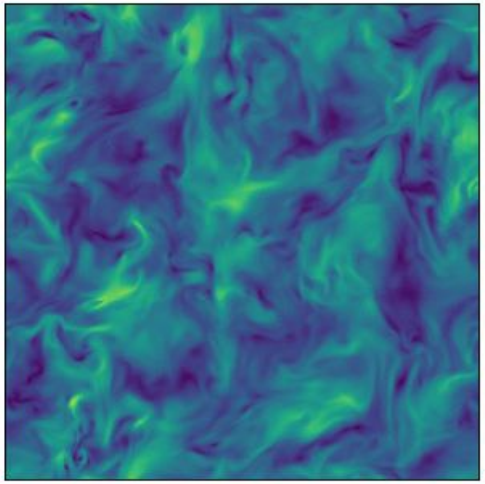
Test Case: Homogenous Isotropic Turbulence (HIT)

- DNS dataset of HIT in a cube – stationary in time. Periodic boundary conditions
- Goal: Learn spatio-temporal 3D dynamics from few snapshots Domain Size: 128^3
- Training Data: 0 – 1 eddy time. Test Data: > 1.5 eddy times.





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Why?

- **Autoencoders are expensive to train for large datasets (e.g. 4096^3 flow)**
- **Interpretable Model reduction is challenging**

Goal: Emulate 3D turbulence more efficiently + better physics intuition/interpretation

Wavelets for Multiscale Datasets

- Locally adaptive, applicable to non-stationary/ aperiodic/ non-linear datasets

- Exploits redundancy in scales turbulence? Multiscale phenomena?

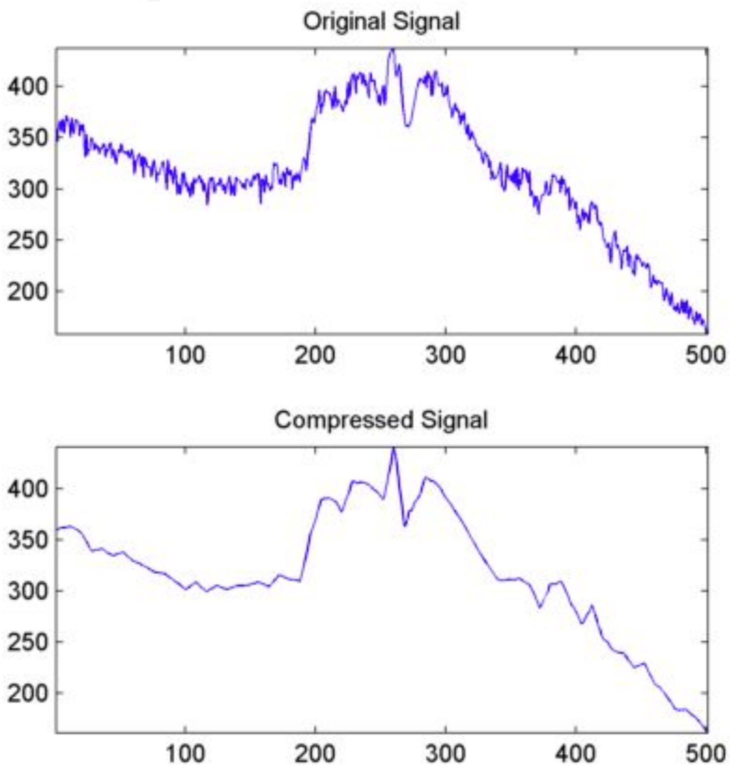
- Several favorable mathematical properties, **can be computed analytically** for any dataset in n-dimensions.

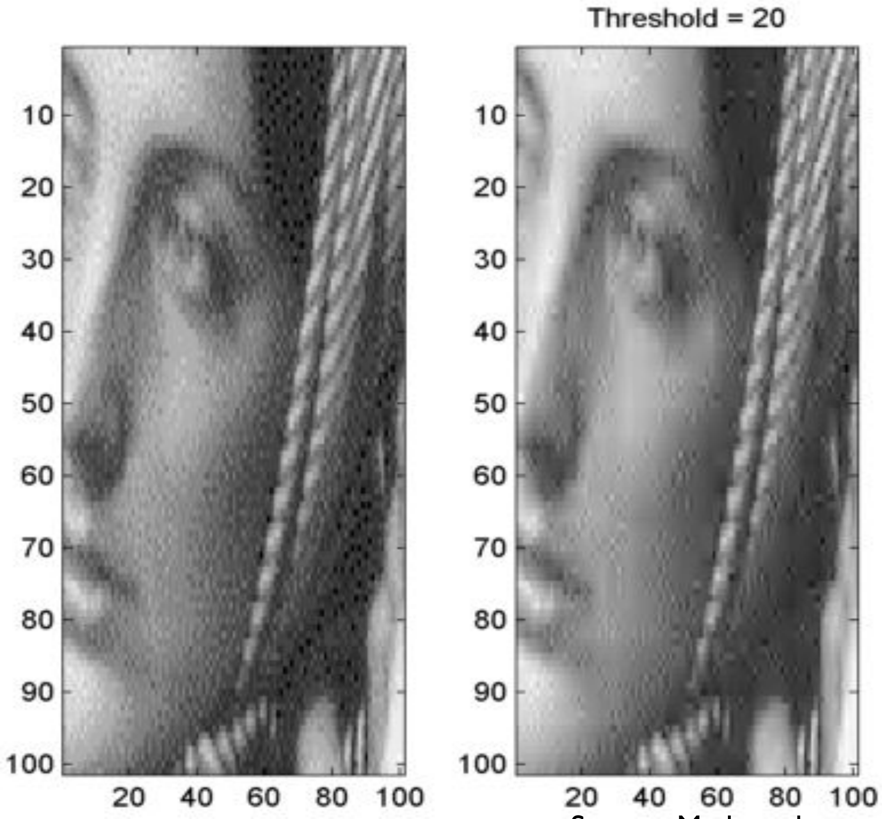
- **Compact representation of information** than raw data can lead to efficient learning.

Excellent candidate for data compression, pattern recognition and reduced order modeling of multi-scale systems – **at low cost**

Wavelet Compression in Action….

<u>Wavelet thresholding</u>: Selecting few coefficients with highest energy, reconstruct the data with the selected i.e. the thresholded wavelets.





Source: Mathworks

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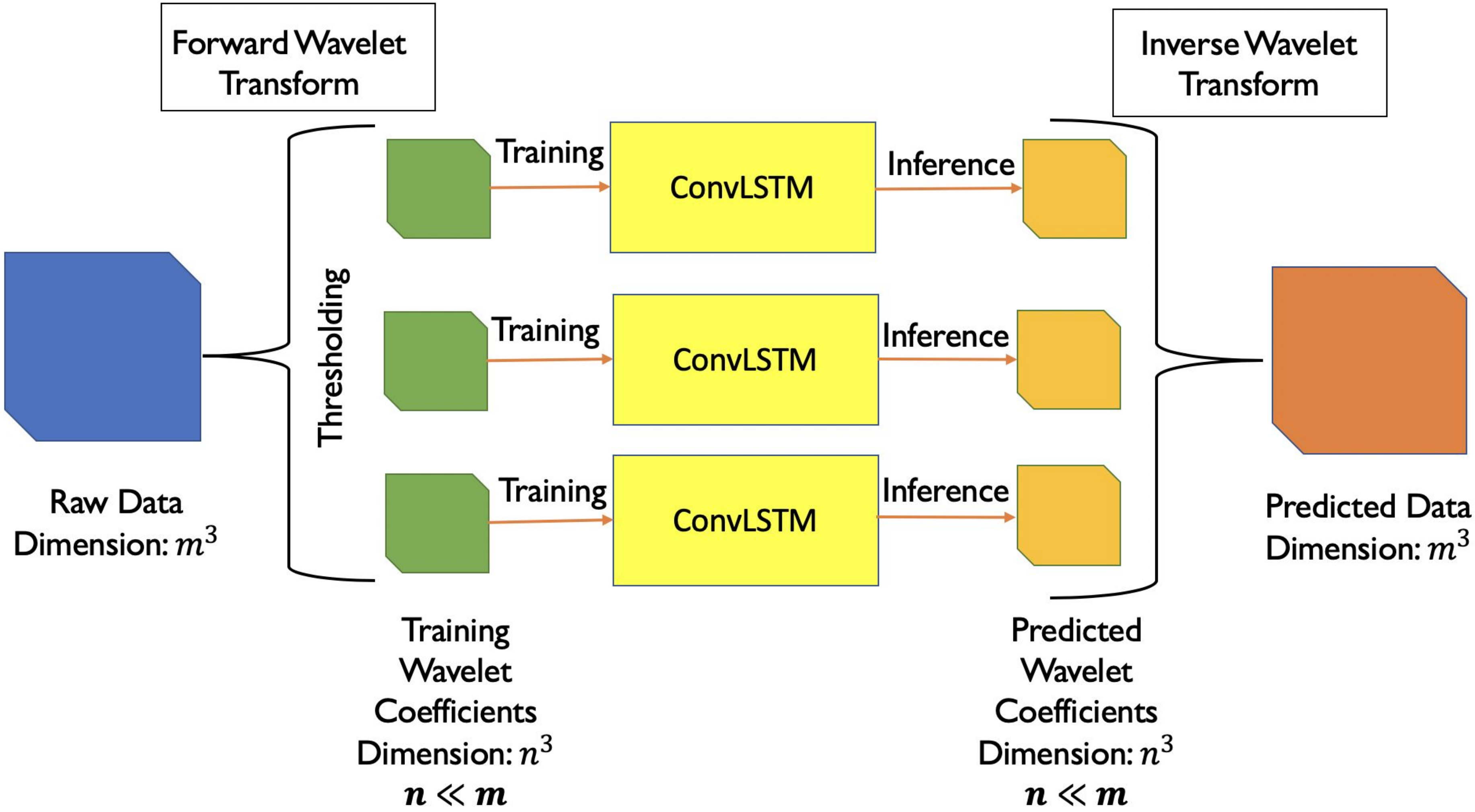
Methodology

Current work: **3%** of wavelet coefficients with highest magnitude chosen. (Each coefficient has 3 velocity components) – Truncate the rest i.e. *Thresholding*

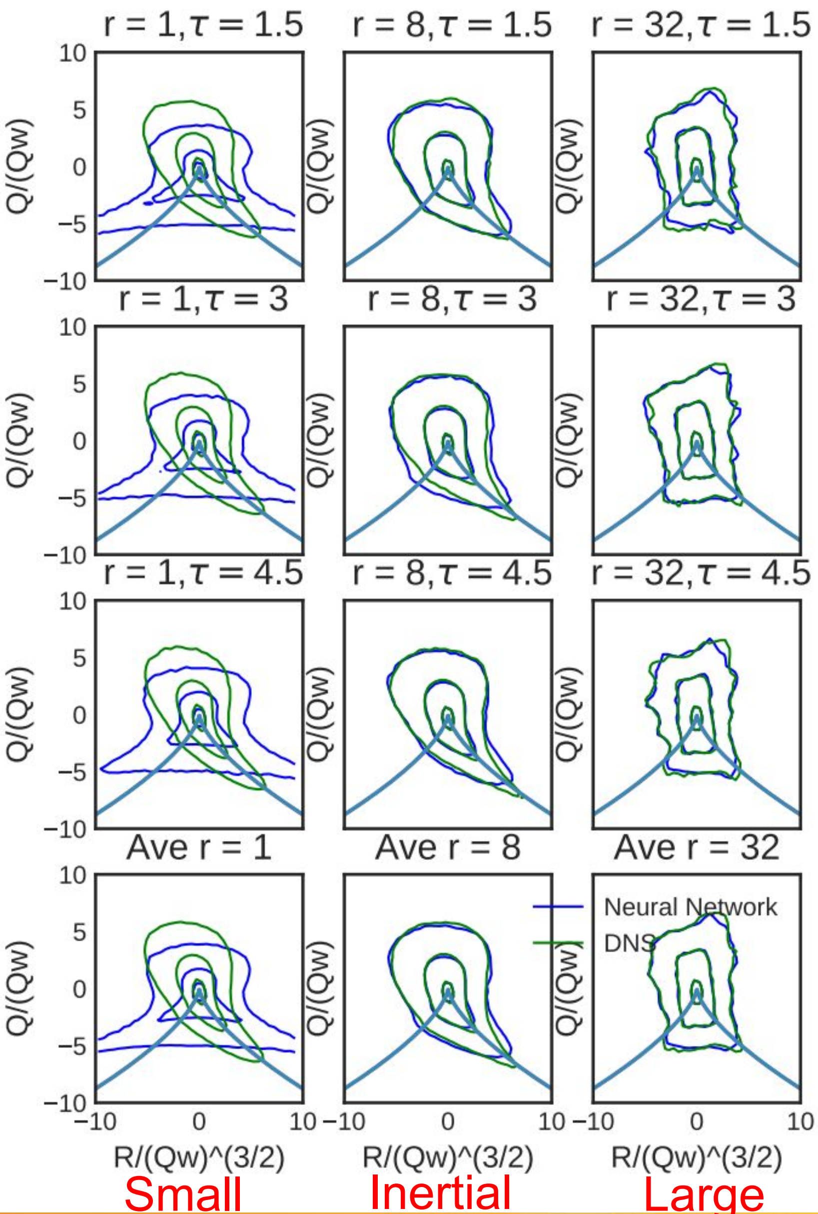
<u>Strategy:</u>

- Decompose velocity field to wavelet space.
- Choose wavelets for thresholding based on energy criteria.
- Train thresholded wavelet coefficients with Convolutional LSTM
- Used learned models to predict wavelet coefficients for future timesteps
- Inverse wavelet transform of all predicted coefficients to obtain velocity field in real space.

Wavelet – Convolutional LSTM



RESULTS



Q-R plane morphology of Small, Inertial and Large Scales – **Most stringent test of 3D turbulence.**

- ✔ Wavelet-CLSTM captures Large scale features very well – lesser accuracy at inertial scales.
- ✔ Errors in small scales due to truncation of coefficients
- ✔ Trained on1.25 eddy times, predictions stable upto 6 □ *Temporally stable predictions.*

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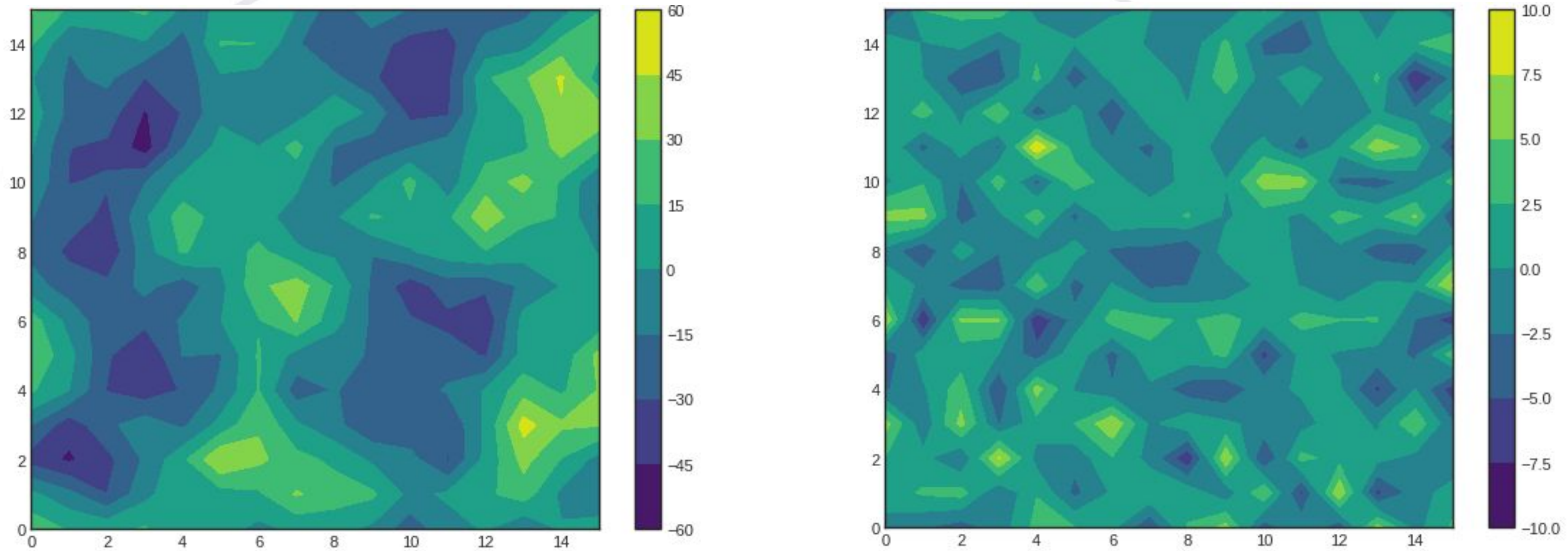
Convolutional Kernel Size is not just A hyperparameter….



Coeff 1 – Highest Magnitude/Large Scales

Kernel (3,3,3) fails. A larger kernel (7,7,7) gives accurate results

Coeff 14 – Low Magnitude/ Small Scales

Kernel (3,3,3) **and** (7,7,7) train well.

Relationship b/w Wavelet Scale size and Conv. Kernel size to build CNNs

Advantages: Wavelet-ConvLSTM

•Analytical representation of wavelets greatly reduces cost. Wavelet thresholding can be studied independently before training a neural network.

•Strong theoretical foundations for wavelets → helpful in interpreting neural network predictions.

•HPC Workload: Training wavelet coefficients is embarrassingly parallel → ZERO inter-node communication overhead due to wavelets being locally adaptive and independent. Can be leveraged for very large datasets.

•Efficient learning: Neural networks learns much faster compared to autoencoder representation → Efficient representation thru spatial redundancy in wavelet basis.

Thank you!

arvindm@lanl.gov

@ArvindMohan15

Rio Grande River Los Alamos, NM